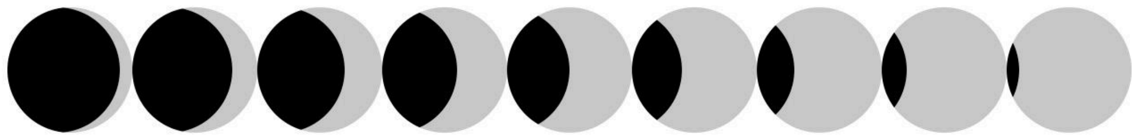
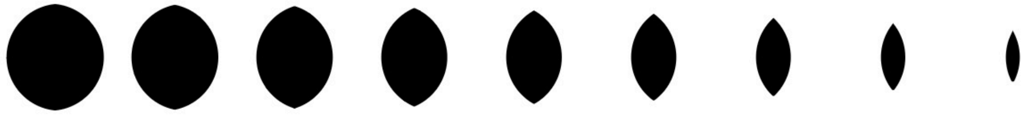


Inquiries-Week 1: Circle Shading

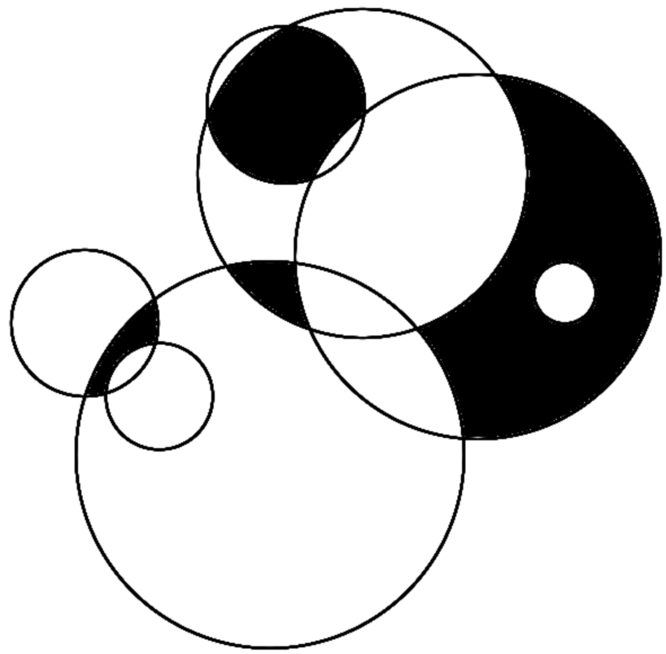
Introduction

When circles overlap they make **lunes:**

or **lenses:**



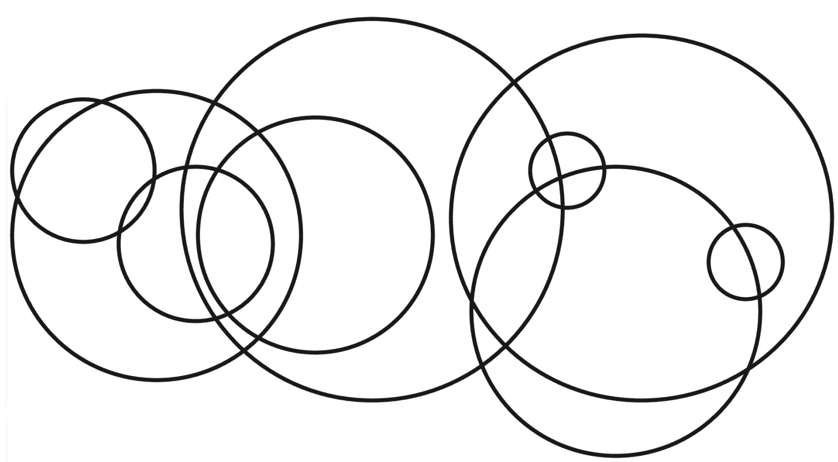
or other fun shapes:



Activity

Let's play with how we color circles that overlap.

First, draw a few circles on a page. **Make their intersections distinct (no overlapping circles).**



Challenge

- Color the regions so no adjacent regions share the same color.
- Find the smallest number of colors needed to color the regions this way.
- Find a rule or algorithm that can be used to determine each region's color.
- Make a conjecture about the rule and test it.
- Bonus: What would a proof look like?

Educator Resources

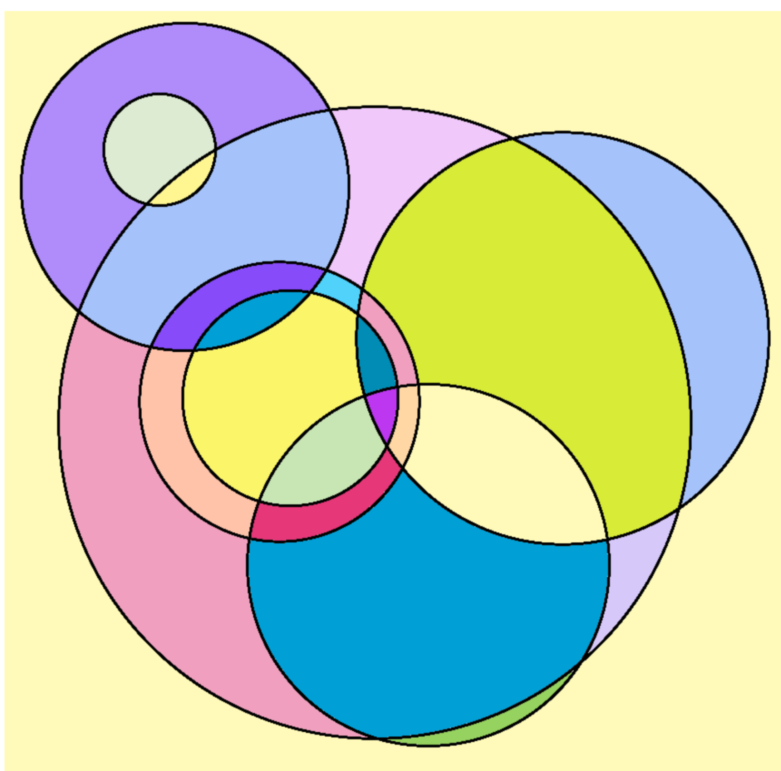
Spoiler alert - go play before proceeding (this means you too).

Activity Structure

This is a 45-60 minute activity to explore developing conjectures with visual patterns.

Exploration Phase (5-10 minutes)

Give time to color and draw circles to form ideas. If there is any confusion on the task - give an example that uses **many** colors: (Here is a tool to play - also full page option).



Conjecture Formation (5-10 minutes)

Allow for time to write down observations and form conjectures. Give examples of conjectures if needed. There are two questions to answer here:

- What is the minimum number of colors needed?
- What rule can be used to determine each region's color?

Example: "A graph of overlapping distinct circles needs ____ colors so that no two adjacent regions have the same color."

Example: "Any region's color can be determined by <insert rule>."

Supporting Questions:

- *"What do you notice about regions that share a boundary?"*
- *"When you cross a circle boundary, what happens to the count of circles containing the region?"* (this question can give it away)
- *"Can you find any arrangement where three colors are necessary?"*
- *"If three or more circles intersect at the same point does that change the results?"*

Discussion and Discovery (10-15 minutes)

- Share conjectures.
- Discuss different approaches.
- Guide learners toward the even-odd insight if they don't discover it.
  - Induction is a good approach for this - start with one circle, then two, etc.
  - Add questions as needed to start thinking of how many circles contain a region.
- Once a two-color conjecture is found, here is a tool to play with parity flipping.

Example Student Conjectures

- "Two colors are needed."
- "The color depends on how many circles contain the region."
- "Two colors are sufficient for coloring regions formed by overlapping circles."
- "Color regions based on whether they are contained in an even or odd number of circles."
- "Overlapping circles can be represented with a bipartite graph."

Possible Misconceptions

- "The number of colors equals the number of regions."
- "Four colors are needed."

Optional - Proof

Overview

The key insight is that as you cross any circle boundary, you either enter or exit exactly one circle. This changes the parity (odd/even) of the number of circles containing that region. Therefore, adjacent regions must have different parities, which means they can be colored with just two colors.

- Here is a tool to show parity flipping and play once the two color conjecture is found.
- Using a whiteboard for each step is useful.

Leading Induction Questions

- With 1 circle, how many regions? How many colors are needed?
- Induction hypothesis: Assume two colors work for n circles.
- Inductive step: What happens when we add circle n+1?
- How does this new circle affect the existing regions?
- Does the coloring rule still work? Why?

Example Proof:

For any arrangement of circles with distinct intersections:

1. *When crossing any circle boundary, we move from one region to an adjacent region.*
2. *The number of circles containing the region changes by exactly ±1.*
3. *Therefore, adjacent regions must have opposite parities.*
4. *If we give even parities one color and odd parities another, no two adjacent regions share a color.*
5. *Therefore, exactly two colors are necessary to color any arrangement of circles with distinct intersections.*

Tools and Supplies

This can be done virtually, on paper, or with code.

- **On paper:**
  - crayons/markers
  - paper
  - compass - optional (circles don't need to be perfect)
- **Digital Drawing tools:**
  - In browser circle paint(on codepen) or on this site here.
  - Online whiteboards (miro, figma, etc) - participants can mark regions color without coloring in all the way
- **Coding - (not a 60 min activity)**
  - Coding is not a single session activity and is more of a prompt to then go contemplate.
  - P5js, python, shaders or other languages can be used to figure out the algorithm.

Vocabulary

- **Lune**: A crescent-shaped region formed when one circle partially overlaps another.
- **Lens**: The almond-shaped region where two circles overlap.
- **Adjacent regions**: Regions that share a boundary segment.
- **Chromatic number**: The minimum number of colors needed to properly color a set of regions.
- **Bipartite graph**: A graph whose vertices can be divided into two groups with no edges connecting vertices within the same group.
- **Parity**: The property of being even or odd.
- **Conjecture**: A mathematical statement that is believed to be true but has not yet been proven.
- **Counterexample**: A specific example that disproves a conjecture.
- **Even-odd rule**: The principle that regions can be colored based on whether they're contained in an even or odd number of circles.
- **Invariant**: A property that remains unchanged under certain operations (such as going from one region to another and changing parity).
- **Boundary**: The line or curve that separates two regions.
- **Intersection**: The place where two or more circles overlap.
- **Region**: A connected area bounded by circle arcs and/or the unbounded exterior.

Extensions and What Ifs

1. **Graph Representation:** Create a graph where each region is a vertex, and regions sharing a boundary are connected by edges. What does this graph look like? How does this relate to the chromatic number? (Bipartite graphs)
2. **Different Shapes:** What happens if we use other shapes instead of circles, such as triangles or squares? Does the minimum number of colors change?
3. **Single Line-Art Shape:** Place a pencil on the paper and draw anything with a single line, distinct intersections, and so that the end and start are the same. How many colors are needed?
4. **How many intersection points can two circles have?**
5. **Make a drawing that needs more than two colors - how is it different?**